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### "We Sold a Million Units" - The Role of Advertising Past-Sales

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# “We sold a million units”-The Role of Advertising Past-Sales.

Paulo Klinger Monteiro and José Luis Moraga-González<sup>✉</sup>

July, 1999

## Abstract

In a market where past-sales embed information about consumers' tastes (quality), we analyze the seller's incentives to invest in a costly advertising campaign to report them under two informational assumptions. In the first scenario, a pooling equilibrium with past-sales advertising is derived. Information revelation only occurs when the seller benefits from the herding behaviour that the advertising campaign induces on the part of consumers. In the second informational regime, a separating equilibrium with past-sales advertising is computed. Information revelation always happens, either through prices or through costly advertisements.

JEL classification: D82, L15, M37

Keywords: two-sided quality uncertainty, past-sales advertising, pooling, signalling, herding behaviour

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# 1 Introduction

In a casual glance to a Sunday's newspaper, one commonly finds advertisements where sellers publicly announce their past-sales. For example, Alfaguara publishers recently inserted an advertisement into Spanish newspaper *El País* where a picture of Javier Marías's novel entitled *Negra espalda del tiempo* appeared together with the following caption: "100.000 copies sold. One hundred thousand possible reasons to read this novel".<sup>1</sup> There are many other instances where one observe such marketing strategies. Pharmaceutical firms often distribute advertisements to report the percentage of doctors or dentists that use certain treatments and health products. Car and motorbike companies frequently invest in publicity to stress that certain model has been the most sold during the previous month or year. Advertising of music records usually emphasize the number of units sold. Managers of theater plays or movies commonly produce advertisements reporting the proceeds obtained, or the number of weeks that they have been performing or on screen. TV and radio programs usually advertise the number (or an estimate) of people who watch or listen to them. Finally, amusement parks<sup>2</sup> and tourism managers repeatedly report the number of tourists who consume their services.

The existence of this class of advertisements generates a number of questions. On the part of the consumers, what should they understand after observing (or not observing) an advertisement of this type? Is the information released useful for the consumers to make wiser decisions? Suppose that the information is useful *ex ante*, does this necessarily mean that buyers will be satisfied *ex post*? On the part of the supply side of the market, one should ask under which conditions a seller has incentives to advertise its privately acquired past-sales information. Does a seller of a moderately demanded product have the same incentives to promulgate its market share than one of a best-seller good? How do these incentives vary with the precision of the exogenous information consumers have?

To analyze these issues, we employ a linear-quadratic-normal<sup>3</sup> two period model<sup>4</sup> similar to Judd and Riordan (1994) whose main features are as fol-

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<sup>1</sup>See *El País*, December 1998.

<sup>2</sup>like EuroDisney (Paris) or Port Aventura (Tarragona, Spain)

<sup>3</sup>normality is not used in the first part of the article.

<sup>4</sup>as those models used in the literature on Information Sharing in Oligopoly (see e.g. Gal-Or (1985, 1986) and Vives (1985)).

lows. A single long-lived seller offers an experience good to two successive infinite generations of consumers with equal tastes. Before the market opens, Nature selects the quality of the good and all consumers privately receive an imperfectly informative signal about the true parameter. The seller receives no valuable information. In the first market opening, the seller, under complete ignorance, sets an initial price and buyers make their demands basing upon the private information they possess (their noisy signals). First-period sales, which are privately observed by the monopolist, thus constitute an aggregate indicator (or a summary statistic) of the good's quality, or equivalently, consumers' tastes. In the subsequent period, the seller sets a price as well as decides whether or not to initiate a costly advertising campaign to report its past-sales. Finally second-generation consumers make their purchases basing upon all information available they have, i.e. their private signals, the observed price and the seen advertisement (if it happens). We assume that advertising is costly and reaches all consumers. Also, we assume that price history is observable.<sup>5</sup>

The analysis is carried out under two scenarios regarding the information available to second-generation buyers. In the first scenario, second-period consumers are completely uninformed about the unknown quality parameter. In contrast, in the second scenario we extend the analysis by allowing for better, but not completely, informed buyers.

Our results are as follows. The first informational scenario, i.e. that in which second-generation buyers are entirely ignorant, is characterized by the fact that prices are incapable to transmit the private information owned by the seller. Thus, the equilibrium exhibits the characteristics of a pooling equilibrium. The equilibrium we derive is however more interesting. We refer to it as a price-pooling equilibrium with past-sales advertising. It consists of two objects: First, a partition of the set of possible sales observations into two subsets: the advertising subset and the no-advertising one. The second object is a pricing function for each subset. If observed sales-data fall into the advertising subset, it pays for the monopolist to invest in ad-

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<sup>5</sup>This is an important assumption. Indeed, it frees our model from signal jamming possibilities (see Caminal and Vives (1996) for a model where firms signal-jam second-period buyers' inferences by quoting appropriate first-period prices). This allows us to concentrate on the second-period decisions and solve the problem separately. All that first-period brings to the analysis is a help to understand the information it provides and its particular nature (past-sales are informative about the unknown taste parameter). This will be useful to discuss some issues later in the paper.

vertising and quote the price that would be charged if there was symmetric information between the seller and the consumers. If sales-data fall into the no-advertising subset, it is optimal for the seller not to invest in advertising and charge the pooling price, which is not informative at all. Of course, consumers are rational and in equilibrium infer the set of sales-data that are not advertised correctly. Therefore, when they do not observe the firm investing in advertisements to report past-sales information, they form the appropriate inferences. Here “no news” means “bad news”. The advertising set is therefore larger than expected due to the seller’s intention to avoid that consumers form inferences that are too pessimistic. Moreover, the lower advertising costs the larger is the advertising set.

The fact that a seller who observes low sales will not voluntarily disclose this event implies that he would benefit ex ante if he could credibly commit to advertising its past-sales. We show, however, that the seller prefers not to commit to release sales information and use the strategy that our advertising equilibrium prescribes.

In the second part of the paper, we turn to an informational regime where second-generation buyers are better informed. In particular, consumers of the second generation receive imperfectly informative signals. This informational scenario is characterized by the fact that prices are capable to signal the private information possessed by the seller. The equilibrium we derive exhibits similar features to the one where prices cannot convey any information. The main characteristic of our price-signalling equilibrium with past-sales advertising is that the seller and the buyers have symmetric information for any past-sales realization. For promising observations, the seller finds it optimal to initiate an advertising campaign to report them, and the price is the one that would be charged if the seller and the buyers had symmetric information. For unpromising sales-data, the seller uses a price-signalling strategy. Moreover, the lower the advertising costs, the smaller is the no-advertising set.

Our research presents several aspects related to the literature on “herding behaviour”.<sup>6</sup> Since individual signals are less accurate than the summary statistic which is embedded in the past-sales, consumers necessarily employ the information released through the advertisements or prices (if signalling occurs). Therefore, it may very well happen that consumers purchase a “lemon” simply because first-generation consumers received good, but wrong,

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<sup>6</sup>See e.g. Banerjee (1992) and Bikhchandani et al. (1992).

signals.<sup>7</sup> This fact, so-called “path-dependence” in the herding literature, is caused by the very fact that buyers do not observe the ex-post utilities of previous customers, but their decisions, which are based on the observed random signals. In our paper, this effect is clearly more accentuated when first-generation buyers are few. Clearly, the idiosyncrasy of the outcomes obtained is less severe when second-generation buyers have corroborating information, as in the second informational scenario of our paper.<sup>8</sup>

The remainder of the paper is organized as follows. Next section describes the model and sets up the problem. The results for the case where second-generation buyers are fully uninformed are presented in Section 3. We extend the analysis to allow for better informed consumers in Section 4. Section 5 concludes.

## 2 The model

We consider a two period economy where there is two-sided uncertainty. A single firm sells a good of uncertain quality  $q$  to two successive generations of consumers.<sup>9</sup> The quality parameter  $q$  is a zero-mean random variable distributed according to the density function  $f(q)$ : In this work, we follow Judd and Riordan (1994) and consider the quality  $q$  as a taste index rather than as a parameter of technical superiority. This perspective allows for the abstraction from the dependency of quality and costs. We thus normalize unit production cost to zero.

A new cohort of  $N$  customers enters the demand side of the market each period.<sup>10</sup> It is assumed that they take the quadratic utility function  $U(x; p; q) = (a + q)x - \frac{x^2}{2} - xp$ ; if they buy  $x$  units of a product of quality  $q$  at unitary price  $p$ : Consumers are short-lived, which implies that they buy only once and that they cannot postpone their purchasing decisions. When the market opens, buyers within a cohort may differ in their information, but

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<sup>7</sup>In the cinema industry this is very common. Even though people agglomerate at the cinemas’ doors to get tickets of new films, occasionally the ex-post valuation of the movie is low. Here this would be a case where realization of  $q$  is low whereas realizations of signals are high.

<sup>8</sup>Bikhchandani et al. (1998) report the success and failure of new products apparently due to no objective reason.

<sup>9</sup>The generalization of our model to more than two periods is immediate.

<sup>10</sup>The number of consumers of each generation could be different. This would not affect our results.

they are all identical ex-ante. Under perfect information, then, the representative consumer of either of the generations would demand  $x = (a + q) / p$ .<sup>11</sup>

The market evolves as follows. At the beginning, before the market opens,  $q$  is drawn by Nature and remains thereafter.<sup>12</sup> Neither firm nor customers receive full information about it. Here, seller's uncertainty is never fully resolved and consumers' uncertainty is resolved ex-post, i.e. after consumption occurs. Once Nature has chosen the taste index, all first-generation customers receive a private signal  $s_1^i$ ;  $i = 1, \dots, N$ ; which conveys (noisy) information about  $q$ . More precisely,  $s_1^i = q + z_1^i$ ; where  $q$  is the realized quality level and  $z_1^i$  is a zero mean random variable with density function  $f(z)$ . One may think of this external information received by consumers to be a result of the natural effort that the firm must exert to introduce the good into the market (e.g. introductory advertising and product demonstrations). Then, under complete ignorance, the seller sets his first-period price and first-generation consumers make their purchases. Once the seller has observed first-period sales, he decides on his marketing strategy, i.e. on the price to be charged to second-generation consumers and on whether or not to invest in an costly advertising campaign to report first-period sales. Finally, second-generation buyers may their purchases.

We analyze two informational scenarios in regard to the second period of market interaction. In Section 3, we study an informational regime where second-generation customers are fully uninformed, i.e. they do not have any external private information valuable. In Section 4, we allow for better informed consumers. There, second-generation buyers receive imperfectly informative signals  $s_2^i = q + z_2^i$ ;  $i = 1, 2, \dots, N$ . Throughout, it is assumed that buyers do not pool their private information, neither within nor between cohorts. Also, we assume that customers observe the history of prices charged, but do not observe quantities sold in the past.

Before proceeding further, some important observations are necessary. The first observation gives the basis to the problem we analyze. Notice that first-generation consumers will demand the good according to their private information. Therefore, realized first-period sales embed a summary statis-

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<sup>11</sup>Later in the paper,  $q$  will be normally distributed. This implies that demand can be negative when  $p > a + q$ . As it is commonly argued in the literature on Information Sharing in Oligopoly, which usually employs linear-quadratic-normal models (see e.g. Vives (1984, footnote 2)), the probability that demand is negative can be made arbitrarily small by choosing the variances of the random variables appropriately.

<sup>12</sup>Notice that this implies that consumers of different generations have equal tastes.



tic of first-generation consumers' tastes, which is private to the seller, and (ex-ante) valuable for all the agents in the marketplace, in order to better estimate the unknown parameter  $q$ . The fact that consumers would be able to make wiser decisions if they observed sales-data raises the question of whether or not the seller is interested in spending resources in an advertising campaign to report its sales.

The second observation has to do with our assumption that the history of prices is observable. Since second-generation consumers observe first-period prices, the seller cannot signal-jam buyers' inferences about the uncertain parameter  $q$  by quoting a particular first-period price. Therefore, our model is a jamming free signal one. This implies that the intertemporal feature of the monopolist's problem does not affect its first-period price decision. As a consequence, we can solve the monopolist problem separately. Next we solve for the seller's first-period decision. Sections 3 and 4 are the core of our analysis where we study the seller second-period marketing strategy.

Let us calculate the first-period demand. First-generation consumer  $i$ 's demand, conditional upon the privately observed signal and any other information available to him, will be

$$x_1^i = a + E[q | \mathcal{I}_1^i] - p_1;$$

where  $\mathcal{I}_1^i$  denotes consumer  $i$ 's information set. Particularly, in the first period,  $\mathcal{I}_1^i = \{s_1^i, p_1\}$ : Note that even though consumers also observe the price charged by the firm in period 1,  $p_1$ , it is not informative at all since the seller does not have any private information on  $q$  at that stage. Therefore  $\mathcal{I}_1^i = \{s_1^i\}$ : We can sum up consumers' demands to obtain the average aggregate demand

$$\bar{X}_1 = a + q_N - p_1;$$

where  $q_N = \frac{1}{N} \sum_{i=1}^N E[q | \mathcal{I}_1^i]$  denotes the average aggregate consumers' expectation about the uncertain parameter  $q$ : Note that, since consumers' signals are private to them and they do not exchange information, realized first-period sales,  $\bar{X}_1$ , are private information to the seller. Consequently, the summary statistic  $q_N$  is known privately by the monopolist. Observe also that  $q_N$  is a random variable and that the seller can improve its inferences on  $q$  by calculating  $E[q | q_N]$ ; which gives a more precise estimator of  $q$  as compared to the prior  $E[q]$ :<sup>13</sup> Clearly, the monopolist may want to condition its second-period marketing strategy on its observation of  $q_N$ :

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<sup>13</sup>To illustrate further, suppose that  $q$  and  $\epsilon^2$  have zero-mean independent normal

The monopolist sets  $p_1$  to maximize his expected short-run profits, i.e. the seller maximizes  $E[\pi_1] = E[(a + q_N - p_1)p_1]$ . The solution of this problem is  $p_1 = \frac{a}{2}$ . Realized sales will then be  $\bar{X}_1 = a + q_N$  and profits  $\pi_1 = a(a + q_N) = 2a^2$ .

Notice that all that is important about the first-period in our model is the information it provides and its nature. Our modelling choice gives more structure and economic intuition to the flow of information in the marketplace. As discussed later in the paper, it is also useful to answer some of the questions posed in the introductory section.

We solve monopolist's second-period problem in what follows. We use the notion of perfect Bayesian equilibrium. This requires consumers' decisions to be optimal given the seller's strategy and their beliefs about  $q$ ; and the seller's strategy to be a best-reply to consumers' actions. Besides, all agents' beliefs must conform to Bayes' rule whenever it applies.

### 3 The basic case: Second-generation buyers are uninformed

We first examine the case where second-generation consumers do not possess any valuable external information. Consequently, they are completely uninformed in advance the seller sets its marketing strategy. This is because buyers do not observe first-period sales. Their prior belief is then  $E[q] = 0$ . This assumption illustrates a situation where buyers know the existence of the product but are completely uninformed about its quality. In Section 4, we extend the analysis to allow for better informed second-period buyers (i.e. they will receive external informative signals).

Next we find the monopolist's optimal prices and profits in the case that he does not invest in an advertising campaign to report its past sales, and in the case which he does. We assume that the cost of the advertising campaign is  $c > 0$ :

distributions with variances  $\frac{1}{N}q^2$  and  $\frac{1}{N}z^2$  respectively. Then, first-period aggregate demand would equal  $\bar{X}_1 = a + \frac{1}{N} \sum_{i=1}^N (q + z_i) - p_1$ ; with  $\frac{1}{N} = \frac{1}{N}(\frac{1}{N}q^2 + \frac{1}{N}z^2)$ . Once sales have been realized, the seller knows  $\bar{X}_1$  and therefore can compute the number  $q_N = (\bar{X}_1 - a + p_1) = \frac{1}{N} \sum_{i=1}^N (q + z_i)$ ; which is an unbiased estimator of  $q$ : In fact,  $q_N$  is a random variable normally distributed with center at zero and variance  $\frac{\frac{1}{N}q^2}{(\frac{1}{N}q^2 + \frac{1}{N}z^2)^2} (\frac{1}{N}q^2 + \frac{1}{N}z^2) = \frac{1}{N}$ :

Suppose that the seller does not invest resources to advertise  $\bar{X}_1$ : Then, in the second period, every buyer's set of information contains only the observed price, i.e.  $- \frac{1}{2} = f p_2 g$  for all  $i$ :<sup>14</sup> As a result, each customer's demand will be identical. Therefore, average aggregate demand will be

$$\bar{X}_2 = a + q_{2N} i p_2; \quad (1)$$

where  $q_{2N} = \frac{1}{N} \sum_{i=1}^N E[q j p_2] = E[q j p_2]$ :

Even though buyers may try to infer the firm's past-sales  $\bar{X}_1$  basing upon the observed price  $p_2$ , in what follows, we show that if the inference rule is Bayesian, then the price is incapable to transmit such an information here. In other words, the optimal price is uncorrelated with  $q_N$  because no inference rule can be part of an equilibrium. To see this, suppose that buyers made inferences according to the rule  $p_2 = \hat{A}(q_N)$ : If the monopolist charges  $\bar{p}$  then aggregate demand is

$$\bar{X}_2 = a + E[q j \hat{A}(q_N) = \bar{p}] i \bar{p}:$$

Therefore, second-period expected profits,  $E[\frac{1}{2} j q_N]$ , do not depend on  $q_N$ :

$$E[\frac{1}{2} j q_N] = (a + E[q j \hat{A}(q_N) = \bar{p}] i \bar{p})\bar{p}; \quad (2)$$

Hence, the optimal price  $\bar{p}$ , i.e. the price that maximizes (2) does not depend on  $q_N$  either, i.e. it is not a random variable. Therefore, in equilibrium  $E[q j p_2] = E[q] = 0$  and  $\bar{p} = a=2$ :

**Lemma 1** Suppose that the seller does not advertise its past-sales. Then, the unique second-period equilibrium price is  $p_2^c = \frac{a}{2}$ ; which gives profits  $\frac{1}{2}^c = \frac{a^2}{4}$ .

Even though customers are rational and sophisticated and therefore, basing upon the observed price, may want to infer the value of  $\bar{X}_1$  (and hence that of  $q_N$ ), we have seen above that no inference rule can be an equilibrium. The reason for this is the typical one in signalling models. If buyers inferred seller's private information from the price, then any "type" of monopolist, by means of his pricing behavior, would have the same incentives to induce an incorrect belief on the part of consumers with the intention to make higher profits. Consumers understand these incentives that any "type" of seller has,

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<sup>14</sup>Notice that although second-period consumers also observe the first-period price, this is not informative since the seller's choice was not contingent on any private information.

and therefore anticipate that if they made purchases according to an expectations rule such as  $q_N = \bar{q}(p_2)$ ; they would be dissatisfied after consuming the good almost surely. As a result, they should expect any quality after observing any price. In equilibrium, consumers will disregard any information conveyed through the price so that their posterior belief equals their prior, for all  $p_2$ . This indeed causes the price to be uncorrelated with the seller's past-sales.

The pooling nature of this equilibrium (price does not depend on  $q_N$ ) here stems from the fact that consumers do not have extra sources of information. In a related paper, Judd and Riordan (1994) demonstrate that when consumers have information of their own, then signaling may occur in equilibrium. In Section 4 we extend our analysis to allow for this possibility.<sup>15</sup>

Suppose now that the monopolist advertises its past-sales. If this happens, buyers calculate  $q_N$  and become as well informed as the firm. In this case, second-period consumers' information set is  $I_2^i = \{q_N\}$  for all  $i$ . Suppose that the firm charges  $\bar{p}$ . Then, consumers' average aggregate belief will be  $q_{2N} = E[q | q_N; p_2 = \bar{p}]$ . Since the price does not add any extra information  $q_{2N} = E[q | q_N]$ . Then average demand will be  $\bar{X} = a + E[q | q_N] \bar{p}$  and the monopolist selects its price  $p_2$  to maximize expected second-period profits

$$E\pi_2 = (a\bar{p} - \bar{p}^2 + \bar{p}E[q | q_N]) \bar{p} \quad c:$$

The optimal price is therefore

$$\bar{p} = \frac{a + E[q | q_N]}{2}.$$

By substituting this price into the profit function, we obtain  $\pi_2 = \bar{p}^2 \bar{c}$ :

**Lemma 2** Suppose that the seller advertises its past-period sales. Then the unique second-period equilibrium price is  $p_2^r = (a + E[q | q_N])/2$  and the optimal second-period profit is  $\pi_2^r = (a + E[q | q_N])^2/4 \bar{c}$ :

<sup>15</sup>In the Industrial Organization literature, however, there are many models where prices charged by fully informed sellers signal qualities. In our model, apart from the reason already mentioned, the absence of both cost asymmetries and repeated purchases impede signalling to emerge. In a single-period model, Bagwell and Riordan (1991) show that separation is achieved in equilibrium when higher quality firms have higher costs of production. But even when cost asymmetries are negligible, a high quality type may distinguish himself from a low quality one whenever repeated purchases play an important role in the market (see Milgrom and Roberts (1986)).

We can establish a comparison between the profits the seller obtains when he invests in advertising to report its past-sales, and the profits he makes when he does not. It is easily seen that  $\frac{1}{4}_2^r > \frac{1}{4}_2^c$  if and only if

$$(a + E[q | q_N])^2 - 4c - a^2 > 0$$

Or

$$4c < 2aE[q | q_N] + E^2[q | q_N] \quad (3)$$

Clearly, whether or not the seller obtains higher second-period profits by advertising past-sales depends on the observed  $\bar{X}_1$ ; or, in other words, on the realizations of  $q_N$ : One might then be tempted to think that the seller will only initiate a publicity campaign to report  $\bar{X}_1$  whenever the inequality in (3) holds, i.e. when  $\frac{1}{4}_2^r > \frac{1}{4}_2^c$ . But if this were so, then rational consumers should take this into account, and whenever they did not observe advertising, they should make the appropriate inferences. This makes the problem very interesting since the mere fact that the firm does not invest in advertising is informative for the consumers: here “no news” means “bad news”.

In what follows we characterize and analyze the existence of what we called pooling equilibrium with past-sales advertising. In words, it consists of a partition of the set of past-sales observations into two subsets: one for which the monopolist finds it optimal to invest in an advertising campaign to report past-sales, and another where the seller prefers to conceal its private information. For each subset, the monopolist employs different pricing rules.

First, we formally define an advertising policy. Then, we define the equilibrium with past-sales advertising and characterize it.

**Definition 1** An advertising policy is a set  $A \subseteq \mathbb{R}$  such that  $\forall q_N \in \mathbb{R} \quad \exists \alpha \in \{0, 1\} \quad \alpha \in A$ .

**Definition 2** An advertising equilibrium is an advertising policy  $A$  and a pricing function  $p(q_N) = p(A)\hat{A}_A(q_N) + p(A^c)\hat{A}_{A^c}(q_N)$ ; where  $\hat{A}_C$  denotes the characteristic function of the set  $C$ ; such that:

- (a)  $p(A)$  (respectively  $p(A^c)$ ) is optimal if  $q_N \in A$  (respectively  $A^c$ ).
- (b) Consumers conjectures about the advertising policy  $A$  are correct.

**Theorem 3** Suppose that the cumulative distribution function of  $q_N$  is strictly increasing. Then there are  $u$  and  $v$  such that the optimal advertising policy is  $A = R \cap (u; v)$ , where

$$E[q | q_N \in (u; v)] = \frac{1}{2};$$

$$u = \frac{1}{2} \left( a + \sqrt{\frac{p}{4c + (a + 1)^2}} \right); \text{ and } v = \frac{1}{2} \left( a + \sqrt{\frac{p}{4c + (a + 1)^2}} \right).$$

**Proof.** Suppose  $B \subseteq R$  is an advertising policy. As we have seen before, if the firm advertises previous sales, the optimal price is  $\bar{p} = \frac{a + E[q | q_N \in B]}{2}$ . If the firm does not advertise and charges  $p$ , consumers infer that  $q_N^{i-1}(B^c)$  occurred and hence the average demand is  $\bar{X}_2 = a + E[q | q_N^{i-1}(B^c)] \leq p$ . Therefore, the optimal price if the firm does not advertise is  $\bar{p} = \frac{a + E[q | q_N^{i-1}(B^c)]}{2}$ . Firm's profits are then

$$\pi_B = \frac{1}{2} \left( \frac{a + E[q | q_N \in B]}{2} \right)^2 \mathbb{1}_B + \frac{1}{2} \left( \frac{a + E[q | q_N^{i-1}(B^c)]}{2} \right)^2 \mathbb{1}_{B^c} \quad (4)$$

To save on notation, write  $x = E[q | q_N \in B]$  and  $y = E[q | q_N^{i-1}(B^c)]$ : The firm will advertise  $q_N$  if and only if  $(a + x)^2 \geq 4c + (a + y)^2$ . Equivalently, whenever  $\frac{a + x}{2} \geq \sqrt{\frac{4c + (a + y)^2}{4}}$ . That is, for all  $x \in R \cap (u; v)$  where  $u = \frac{1}{2} \left( a + \sqrt{\frac{p}{4c + (a + 1)^2}} \right)$  and  $v = \frac{1}{2} \left( a + \sqrt{\frac{p}{4c + (a + 1)^2}} \right)$ . Therefore  $B$  is optimal if and only if

$$B = R \cap \left( \frac{1}{2} \left( a + \sqrt{\frac{p}{4c + (a + 1)^2}} \right); \frac{1}{2} \left( a + \sqrt{\frac{p}{4c + (a + 1)^2}} \right) \right)$$

and

$$y = E[q | q_N \in (u; v)] = \frac{1}{2} \left( a + \sqrt{\frac{p}{4c + (a + 1)^2}} \right).$$

The last equation allows us to determine  $y$ . Define  $u = \frac{1}{2} \left( a + \sqrt{\frac{p}{4c + (a + 1)^2}} \right)$  and  $v = \frac{1}{2} \left( a + \sqrt{\frac{p}{4c + (a + 1)^2}} \right)$ . Then,  $y$  must solve the implicit equation

$$y = \frac{\int_R q \, dP(q)}{\int_R \mathbb{1}_{(u; v)}(q) \, dP(q)} \quad (5)$$

We may substitute  $E[q | q_N \in (u; v)]$  for  $q$  if desired. To prove the existence of such a  $y$  consider the function  $g : R \rightarrow R$ ,

$$g(y) = \frac{\int_R \frac{1}{2} \left( a + \sqrt{\frac{p}{4c + (a + 1)^2}} \right) \, dP(q)}{\int_R \mathbb{1}_{(u; v)}(q) \, dP(q)} - y.$$

The denominator is never zero since the distribution of  $q_N$  is strictly increasing. Therefore  $g(c)$  is a continuous function. Since  $\lim_{y \rightarrow 1} g(y) = 1$  and  $\lim_{y \rightarrow 0} g(y) = 0$  there is a  $c^*$  such that  $g(c^*) = 0$ . Q.E.D.

**Remark 1** The solution for  $c = 0$  is delicate since the set  $A^c$  will be a zero measure set. Then Bayes' rule is not well defined when  $A^c$  occurs. If consumers for instance consider that  $E[q | q_N] = 1$  when they do not observe past-sales advertising, then, in equilibrium, the firm almost always advertises.

Figure 1 illustrates Theorem 3. In equilibrium, the seller finds it optimal to invest in past-sales advertising whenever  $q_N$  lies in the set A: If this occurs, the accompanying price obviously equals the price that he would charge in the case that there was symmetric information in the market, i.e.  $p = (a + E[q | q_N]) = 2$ . Otherwise, the seller gains by concealing his private information (subset B): In such a case, the accompanying price is a pooling price in the sense that any "type" of seller not investing in advertising charges the same price, i.e.  $p = (a + E[q | q_N^{-1}(B)]) = 2$ : Notice that values of  $q_N$  falling at the left of  $u$  imply that demand is probably negative. Since  $E[q | q_N]$  is a monotonic function of  $q_N$ ; the monopolist initiates the advertising campaign whenever he believes that the good is of relatively high quality. Notice also that the no-advertising set is non-empty provided that advertising cost is positive. Therefore, costly full revelation never occurs.

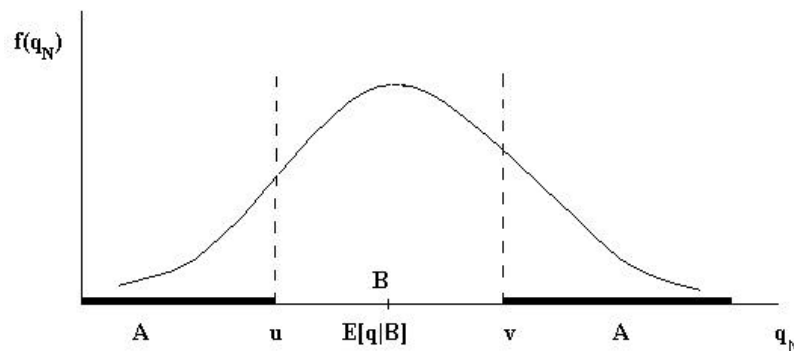
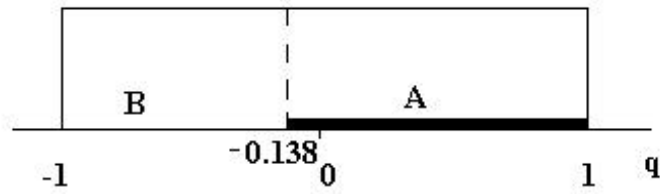


Figure 1: Past-sales advertising (A) and no-advertising (B) subsets

The following examples illustrate the necessary calculations to obtain  $c^*$ ;  $u$  and  $v$ : Let us suppose that  $q$  is uniformly distributed in  $[0, 1]$ . Further,



Example 2:  $a=5, c=1$

Figure 1: Figure 2: Advertising (A) and no-advertising (B) subsets.

suppose that first-generation consumers signals are such that  $q_N = q$ ; i.e. the seller is fully informed about  $q$  after first-period. From equation (5) we obtain

$$\begin{aligned} \theta^1 &= \frac{\int_{\min f_1; v g}^{\max f_i; 1; u g} x dx}{\min f_1; v g + \max f_i; 1; u g} = \frac{\min f_1; v g^2 + \max f_i; 1; u g^2}{2(\min f_1; v g + \max f_i; 1; u g)} \\ &= \frac{\min f_1; v g + \max f_i; 1; u g}{2}. \end{aligned}$$

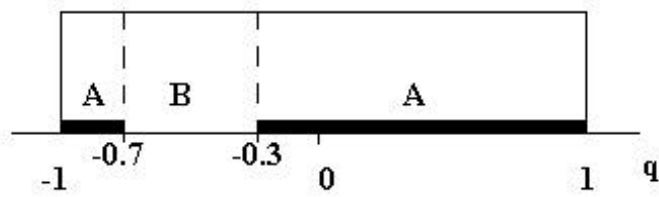
Consider the following examples.

Example 1: If  $4c \geq 1 + 2a$  we have that  $\theta^1 = 0$ ,  $u = \frac{a}{\sqrt{4c + a^2}}$  and  $v = \frac{a}{\sqrt{4c + a^2}} - 1$  are solutions. Thus, in this case advertising never occurs. The reason is that publicity is too costly here.

Example 2: Let us suppose now that  $4c < 1 + 2a$  and for definiteness that  $a > 1$ : Then  $u = 1$ ;  $v = \frac{1}{a} - \frac{2a + 2}{(1 - a)^2 + 12c} = \frac{3}{2} - \frac{1}{a}$  and  $\theta^1 = (v + 1)/2$ . If  $a = 5$  and  $c = 1$ , advertising occurs if  $q > 0.1389$  (see Figure 2).

Example 3: If  $a = 0.5$  and  $c = 0.01$ , then we have that advertising occurs for all  $q \in [1/3; 1]$  (see Figure 3). Notice that the inferences consumers form when they do not observe the publicity campaign are  $E[q] = 0.5$ : For past-sales observations falling in the set  $[0.5; 0.3]$ , the seller does not initiate an advertising campaign because it is too costly, i.e. the benefits obtained from inducing the right belief to the consumers





**Example 3:  $a=0.5, c=0.01$**

Figure 2: Figure 3: Advertising (A) and no-advertising (B) subsets.

do not compensate for the advertising cost. To the left of  $\frac{1}{2}$  consumers beliefs benefit the seller and consequently advertising does not occur.

Observe that in general the set of events for which advertising occurs shrinks as  $c$  increases. In fact, if  $c$  is very large relative to  $a$ , advertising never occurs.

A natural question arises now. Suppose the seller could commit to release its private information before observing it. Would he do it or rather prefer to decide after observing the performance of its good in the market? The following result states the seller prefers our pooling equilibrium with past-sales advertising.

**Theorem 4** The seller's expected profit is higher when he waits to observe his first-period sales and makes his advertising decision contingent on this observation, as compared to the profits that he obtains when he commits to deliver past-sales information at the commencement of the market interaction.

**Proof.** We need to compare expected profits when the firm commits to reveal its past-sales, i.e.

$$\pi_2^c = E \left[ \frac{\mu_a + E[q | q_N]}{2} \right] \pi_2 \quad \text{if } c$$

with the expected profit obtained by employing the strategy prescribed by our pooling equilibrium with past-sales advertising. If this latter case, expected profit is, using (4):

$$\frac{1}{4}^w = E \left[ \frac{\mu_{a + E[q | q_N]} \pi_2}{2} \mathbb{1}_{i \in \hat{A}_B} + \frac{\mu_{a + E[q | B^c]} \pi_2}{2} \mathbb{1}_{\hat{A}_{B^c}} \right]$$

where  $B$  is the optimal advertising set. We see from the proof of theorem 3 that

$$B = \left\{ i \in \mathcal{I} : \frac{\mu_{a + E[q | q_N]} \pi_2}{2} \geq \frac{\mu_{a + E[q | B^c]} \pi_2}{2} \right\}$$

Thus we have that

$$\begin{aligned} \frac{1}{4}^w &= E \max \left( \frac{\mu_{a + E[q | q_N]} \pi_2}{2} \mathbb{1}_{i \in B}; \frac{\mu_{a + E[q | B^c]} \pi_2}{2} \mathbb{1}_{i \in B^c} \right) \\ &= E \frac{\mu_{a + E[q | q_N]} \pi_2}{2} \mathbb{1}_{i \in B} = \frac{1}{4}_2^r. \end{aligned}$$

Q.E.D. .

In our model, “herding” on the part of the consumers necessarily occurs. It happens as a result of their rational behaviour: buyers always want to employ the information released because it allows them to compute better estimates of  $q$ . In equilibrium, the seller anticipates consumers’ behavioral rules and chooses its marketing strategy accordingly. When seller observes that its product is not a best-seller, he prefers to avoid buyers’ herding and therefore conceals past-sales information. Otherwise, he is interested in helping herding to occur and does so by initiating a publicity campaign to report its past-sales.

As it is common in models of herding behaviour, the equilibrium outcomes here are also idiosyncratic and exhibit path-dependence. This means that even though it is ex-ante optimal for the buyers to use the information revealed through the advertisements, buyers may be dissatisfied ex-post consumption. Imagine, for instance, that the drawn  $q$  is low and the realized noise is biased toward positive and high values. First-generation consumers will demand much and so will second-generation buyers after knowing past-sales. At the time to make decisions, second-period consumers cannot do

better than “following the crowd”. However, their decisions will turn to be wrong ex-post consumption. It is well known that the success or failure of new products may very well depend on non-controllable market forces such as consumers external signals.<sup>16</sup> In our pooling equilibrium with past-sales advertising, the seller exploits the information asymmetry in its own interest by letting consumers to herd or not.

In what follows, we extend the analysis to allow for better informed second-generation consumers. Unfortunately, we have not been able to carry out an analysis as general as before regarding the distribution functions of the random variables. From now on, we assume that all variables are normally and independently distributed as follows:  $q \gg N(0; \frac{3}{4}q^2)$ ;  $z_1^i \gg N(0; \frac{3}{4}z_1^2)$ ;  $z_2^i \gg N(0; \frac{3}{4}z_2^2)$ : According to this, the private information to the firm after first-period interaction is  $q_N = \pm_1(q + \frac{1}{N} \sum_{i=1}^N z_1^i)$ ; where  $\pm_1 = \frac{\frac{3}{4}q^2}{\frac{3}{4}q^2 + \frac{3}{4}z_1^2}$ :

## 4 Extension: Second-generation consumers are better informed.

The purpose of this Section is to test the robustness of our results when consumers are better informed. In the preceding Section, we have analyzed a situation where prices are incapable to transmit any information. Here, we turn to a framework where prices can signal past-sales information.

Assume that second-period consumers exogenously receive extra information about the unknown quality parameter through the signals  $s_2^i = q + z_2^i$ ;  $i = 1; \dots; N$ : As Judd and Riordan (1994) show, a signalling equilibrium does exist when consumers have an extra piece of information. Signalling emerges due to the fact that buyers have corroborating information of their own (here their private signals). In what follows, we characterize such an equilibrium. We proceed following the same steps as above.

Suppose first that the monopolist conceals the value of  $\bar{X}_1$ . Then, consumer  $i$ 's second-period information set is  $\mathcal{I}_2^i = \{s_2^i; p_2\}$ ;  $i = 1; \dots; N$ : Suppose the firm charges  $p$ : Then second-generation consumer  $i$ 's demand will be

$$x_2^i = a + E[q | s_2^i; p_2 = p] - p$$

<sup>16</sup>A restaurant may be crowded certain day while another restaurant located just around the corner is relatively empty. The next day, however, the opposite may very well happen (see Bikhchandani et al. (1998) for other examples).

As in the previous section, consumers may try to infer past-sales upon the observed price. As it is usual in linear-normal-quadratic models, we focus on the case where agents employ linear rules for their decisions.<sup>17</sup> Suppose that consumers make inferences using the linear rule  $p_2 = \bar{p} + \bar{v}q_N$ ;  $\bar{v} \in \mathbb{R}$ . Since all variables are normally distributed, there are  $u$  and  $v$  such that  $E[q_j | s_2^i; q_N] = us_2^i + vq_N$ .<sup>18</sup> In addition, there is  $h$  such that  $E[s_2^i | q_N] = E[q_j | q_N] = hq_N$ .<sup>19</sup> The following result will be useful in what follows.

**Lemma 5** It is true that  $uh + v = h$ :

**Proof.**  $E[q_j | q_N] = E[E[q_j | s_2^i; q_N] | q_N] = E[us_2^i + vq_N | q_N] = uE[s_2^i | q_N] + vq_N = uE[q_j | q_N] + vq_N$ . Thus  $E[q_j | q_N] = v/(1 - u)q_N = hq_N$ . Hence  $v = (1 - u)h$  ending the proof. Q.E.D.

With the help of this Lemma we can compute the average aggregate demand:

$$\bar{X}_2 = a + \frac{u}{N} \sum_{i=1}^N s_2^i + v \frac{\bar{p} - \bar{p}^*}{1 - v}.$$

The firm maximizes expected profits conditional on its information set  $I_2 = \{\bar{X}_1\} = \{q_N\}$ ; that is

$$E\pi_2 = \bar{p} - a + uE[q_j | q_N] + v \frac{\bar{p} - \bar{p}^*}{1 - v}.$$

Therefore, the optimal price is

$$p^c = \frac{a + uhq_N + \bar{p}^*}{2(1 - v)}.$$

Since the customers' inference rule must be correct in equilibrium, it must be the case that

$$\bar{p}^* = \frac{a + \bar{p}^*}{2(1 - v)} \text{ and } \bar{v} = \frac{uh}{2(1 - v)}. \quad (6)$$

<sup>17</sup>See e.g. Judd and Riordan (1994).

<sup>18</sup>Explicitly,  $u = \frac{\frac{1}{N} \sum_{i=1}^N s_2^i}{\frac{1}{N} \sum_{i=1}^N s_2^i + N \frac{1}{N} \sum_{i=1}^N s_2^i + \frac{1}{N} \sum_{i=1}^N s_2^i}$  and  $v = \frac{u \frac{1}{N} \sum_{i=1}^N s_2^i}{\frac{1}{N} \sum_{i=1}^N s_2^i}$ .

<sup>19</sup>Namely,  $h = \frac{\frac{1}{N} \sum_{i=1}^N s_2^i}{\frac{1}{N} \sum_{i=1}^N s_2^i + \frac{1}{N} \sum_{i=1}^N s_2^i}$ .

Solving the preceding system of equations (6) we obtain  $\bar{p} = (h + v)/2$  and  $\bar{q} = a(h + v)/2h$ . So the optimal pricing rule is

$$p_2^c = \frac{a(h + v)}{2h} + \frac{h + v}{2}q_N = \frac{h + v}{2h}(a + hq_N): \quad (7)$$

Equilibrium profits are easily computed:

$$\pi_2^c = \frac{uh}{h + v}(p_2^c)^2 = \frac{u(h + v)}{4h}(a + hq_N)^2: \quad (8)$$

The following lemma summarizes:

**Lemma 6** Suppose second-generation consumers exogenously receive informative signals  $s_2^i = q + \varepsilon_2^i$ ;  $i = 1, \dots, N$ . Suppose also that the seller does not advertise his past-sales. Then, there exists a linear separating equilibrium where the firm charges the price given by (7) and obtains profits given by (8).

Notice that the coefficient of  $q_N$  is positive, i.e.  $h + v > 0$ . This means that the price is positively correlated to past-sales. The higher the observation of  $q_N$  (hence the firm's estimation of  $q$ ), the higher is the price charged in the separating equilibrium. Since consumers' inference rule is correct in equilibrium, higher prices signal greater past-sales, and hence higher firm's expected quality.<sup>20</sup>

We now investigate the optimal pricing rule and profits when the firm invests in an advertising campaign to report its past-sales. When consumers are informed about the value of  $\bar{X}_1$ , they can compute  $q_N$ . Therefore, on average, they will demand

$$\bar{X}_2 = a + \frac{u}{N} \sum_{i=1}^N s_2^i + vq_N \text{ if } p_2:$$

Expected profits will be

$$E\pi_2 = p_2(a + uhq_N + vq_N \text{ if } p_2) \text{ if } c = p_2(a + hq_N \text{ if } p_2) \text{ if } c$$

<sup>20</sup>An interesting observation is that when  $\pi_2^c$  approaches infinity, i.e. when second-period signals become less and less informative, the coefficient of  $q_N$  converges to zero. This would be the case where price is uncorrelated to quality, i.e. a pooling situation.

since  $uh + v = h$  (see Lemma 5). The equilibrium price maximizes the previous expression and is therefore<sup>21</sup>

$$p_2^r = \frac{a + hq_N}{2} \quad (9)$$

As before, parameters  $\theta$  and  $\gamma$  must be such that  $\theta = a=2$  and  $\gamma = h=2$ : Equilibrium profits are in this case

$$\pi_2^r = (p_2^r)^2 = \frac{(a + hq_N)^2}{4} \geq c \quad (10)$$

The following Lemma summarizes:

**Lemma 7** Suppose that the seller advertises its past-period sales. Then the unique second-period equilibrium price is  $p_2^r = (a + hq_N)/2$  and the optimal second-period profit is  $\pi_2^r = (a + hq_N)^2/4 \geq c$ :

We are now in a position to study under which conditions it pays for the seller to initiate an advertising campaign to report its sales. Essentially, in the case under consideration, the seller possesses two alternative methods to reveal past-sales. One involves signalling activities. The other involves an advertising campaign. Intuitively, the seller will select the cheapest device among the possible ones. The fact that in a separating equilibrium consumers correctly infer seller's past-sales considerably simplify the computation of our separating equilibrium with past-sales advertising. Indeed, if the firm advertises its past-sales, the optimal price is  $p^r = (a + hq_N)/2$ : On the contrary, if sales information is not delivered, consumers suppose that the pricing function is  $p^c = u(h + v)(a + hq_N)/2h$ : In both cases, consumers are fully informed. Then, the firm will simply initiate a publicity campaign to report past-sales if and only if doing so is cheaper than signalling through price, i.e.  $\pi_2^r \geq c \geq \pi_2^c$ : This inequality amounts to

$$(a + hq_N)^2 \geq 4c + \frac{u(h + v)}{h}(a + hq_N)^2:$$

Collecting terms and simplifying, we obtain that advertising occurs if  $q_N$  does not lie on the set

$$\left\{ \frac{a}{h} \pm 2 \sqrt{\frac{c}{h(h(1 - u) - uv)}}; \frac{a}{h} + 2 \sqrt{\frac{c}{h(h(1 - u) - uv)}} \right\} \quad (11)$$

<sup>21</sup>Notice that  $p_2^r < p_2^c$  for all  $q_N > 0$ : This fact exhibits the usual upward price distortion occurring in separating equilibria.

**Theorem 8** Suppose second-generation consumers exogenously receive informative signals  $s_2^i = q + \frac{1}{2}i$ ;  $i = 1, \dots, N$ : Then there is a signalling equilibrium with past-sales advertising where (a) the seller advertises  $\bar{X}_1$  if  $q_N$  does not fall in set 11 and charges price 9, and (b) the seller conceals the value of  $\bar{X}_1$  if  $q_N$  lies on set 11 and charges the separating price 7.

Figure 4 illustrates this result. For the following parameter values  $a = 20$ ;  $N = 2$ ;  $c = 300$ ;  $\frac{1}{4}q_1^2 = 5$ ;  $\frac{1}{4}q_1^2 = 1$ ; and  $\frac{1}{4}q_2^2 = 1$ ; the thinner line represents the profits obtained when the seller invest in an advertising campaign to report its sales. The thicker line represents the firm's profits when signalling occurs. It is clear that for high values of past-sales observations, it pays for the seller to advertise rather than employ the price as the device to transmit past-sales information. The intuition is that the necessary price distortion to do the latter is higher the greater is  $q_N$ : Indeed, the distortion is  $p_2^c - p_2^f = v(a + hq_N) = 2h > 0$ ; which increases with  $q_N$ : On the other hand, if  $q_N$  is low, then it pays for the seller not to invest in advertising and use the price as a signalling device. Our separating equilibrium with past-sales advertising gives the highest attainable profits to the seller: if past-sales observations fall to the right of the point where the two profit curves cross, advertising occurs. Otherwise, signalling happens.<sup>22</sup>

Again, in our signalling equilibrium with past-sales advertising herding occurs. Irrespective of whether advertising happens or not, in equilibrium, consumers are always informed about the average taste of their predecessors, which induces rational herding behaviour on the part of the consumers. However, since here buyers have corroborating information of their own, the effects of herding are more moderate.

## 5 Conclusions

We would like to answer the questions posed in the introductory section to conclude this paper. In a market where a producer sells a commodity to different generations of consumers who exhibit similar tastes, past-sales typically contain (noisy) information about product's quality. In such environments, we have shown that a monopolist has occasionally an incentive to

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<sup>22</sup>For completeness, we give in the appendix the conditions under which the seller would commit to advertise its past-sales supposing that he had to commit either to release its information or to conceal it.

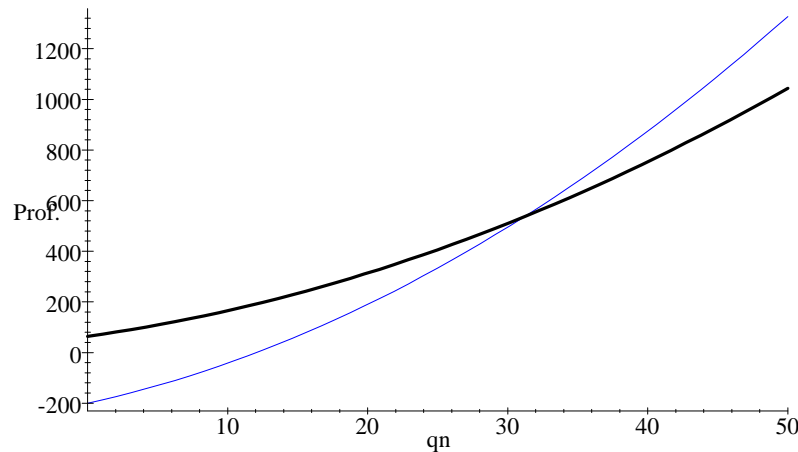


Figure 3: equilibrium with past-sales advertising

invest in advertising activities to report its private sales-data. By doing so, the supplier allows consumers to better estimate the quality of the good, and hence make wiser decisions. Often, but not always, the seller benefits from more clever decisions on the part of the consumers. The equilibrium we derive presents this feature. Indeed advertising occurs for some past-sales observations. When this happens, the equilibrium price is the one which would be charged by the seller if there was symmetric information in the market. When advertising does not occur, either separating prices or pooling prices happen, depending on whether or not consumers have external information of their own.

The information released through the publicity campaign, when it occurs, is ex-ante useful for the consumers. Knowing the average taste of the “parents” allow the “children” to calculate better the unknown parameter. Does this necessarily mean that the children will make the right decisions? Observe that past-sales advertising embeds a summary statistic of the actions taken by the predecessors, therefore not conveying information about actual utilities derived by the consumers. As a consequence, consumers may be dissatisfied ex-post consumption. Of course, this effect, which is due to the rational herding behaviour that occurs on the part of the consumers when past-sales advertising is observed, is more moderate when the children have



information of their own.

## 6 Appendix

For completeness, we analyze here under which conditions a seller would benefit from committing to keeping private its past-sales information (if commitment possibilities are available). We do so for both the informational scenario of Section 3 and for the case where buyers are better informed.

In the Scenario where consumers do not receive any external information,

**Proposition 9** Suppose that the seller could commit either to reveal or to conceal his past-sales. Then, the monopolist would commit to deliver its sales-data if and only if  $4c < E[E^2[q | q_N]]$ :

**Proof.** First, note that first-period profits are identical since in both situations the seller is completely uninformed. Consequently we can abstract from them. By concealing, the seller obtains expected profits  $E[\pi_2^c] = \frac{a^2}{4}$ . By revealing past-sales, expected benefits are  $E[\pi_2^r] = E\left[\frac{a + E[q|q_N]}{2}\right]^2 - c = \frac{a^2}{4} + \frac{E[E^2[q|q_N]]}{4} - c$ . By establishing a comparison between optimal profits in the two cases, the theorem follows. Q.E.D.

**Remark 2** Note that  $E[E^2[q | q_N]] > 0$ : If  $E[q|q_N] \neq 0$  this is immediate. Suppose now that  $E[q|q_N] = 0$ : Since  $q$  and  $z_N$  are independent, we have that  $E[q^2_N] = E[q]E[z_N^2] = 0$ . Thus  $E[q^2] = E[q(q + z_N)] = E[qq_N] = E[E[q|q_N]q_N] = 0$  a contradiction.<sup>23</sup>

To gain some intuition about the condition in Proposition 9, let us look at the case where random variables are normally distributed. The condition reduces to  $4c < \frac{3\sigma_q^4}{\sigma_q^2 + \sigma_z^2} = \frac{3\sigma_q^4}{N}$ : Therefore, it tends to be satisfied when the number of consumers is high, and the noise of the informative signals and advertising cost is low.

In the information Scenario where consumers receive signals of Section 4,

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<sup>23</sup>We are indebted to Claudio Landim, who showed us this compact way of proving it.

**Proposition 10** Suppose that the seller could commit either to reveal or conceal his past-sales. Then, the seller would commit to advertise its past-sales if and only if parameters satisfy

$$4c < a^2 \frac{v(1-u)}{h} + \frac{1}{4}v(1-u):$$

**Proof.** By concealing past sales information the seller obtains expected benefits  $E[\frac{1}{4}c] = E \frac{uh}{h+v} (p_2^c)^2 = \frac{uh(h+v)}{4} \frac{a^2}{h^2} + E[q_N^2]$ . By revealing, the monopolist gets expected profits  $E[\frac{1}{4}c] = E (p_2^r)^2$  if  $c = E \frac{h}{h+uq_N} \frac{a^2}{2}$  if  $c$ : By comparing,  $E[\frac{1}{4}c]$  and  $E[\frac{1}{4}c]$  and using the fact that  $E[q_N^2] = \frac{1}{4} \frac{a^2}{h}$ ; the theorem follows. Q.E.D.

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